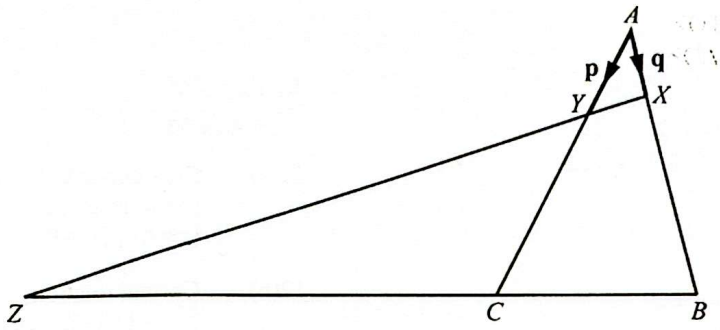
VECTOR GEOMETRY

Q25/11/M/J/16

1.



In the diagram,

X is the point on AB where $AX = \frac{1}{4}AB$,

Y is the point on AC where $AY = \frac{1}{3}AC$,

Z is the point on BC produced where $CZ = 2BC$.

$\overrightarrow{AY} = \mathbf{p}$ and $\overrightarrow{AX} = \mathbf{q}$.

(a) Express, as simply as possible, in terms of **p** and **q**,

(i) $\overrightarrow{XY}$,

Answer $\overrightarrow{XY}$ = ... [1]

(ii) $\overrightarrow{BC}$,

Answer $\overrightarrow{BC}$ = ... [1]

(iii) $\overrightarrow{XZ}$.

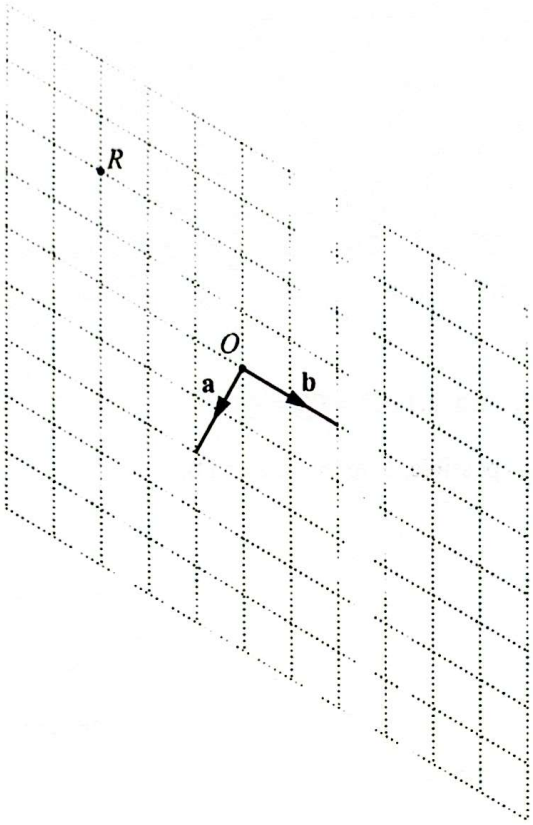
Answer $\overrightarrow{XZ}$ = ... [2]

(b) Hence find $XY : YZ$.

Answer ..... : ..... [1]

Q19/11/O/N/16

2.



The diagram shows the points O and R and the vectors **a** and **b**.

(a) Given that $\overrightarrow{OP} = 2\mathbf{a}$, mark and label the position of P on the grid. [1]

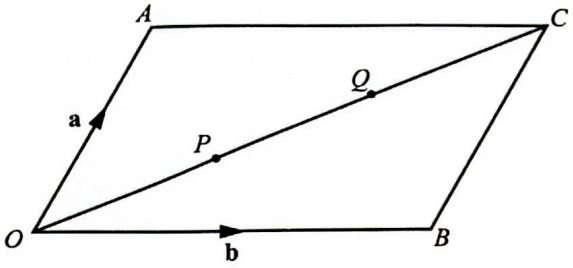
(b) Given that $\overrightarrow{OQ} = 2\mathbf{b} - \mathbf{a}$, mark and label the position of Q on the grid. [1]

(c) Express $\overrightarrow{OR}$ in terms of **a** and **b**.

Answer $\overrightarrow{OR}$ = ... [2]

Q21/12/M/J/17

3.



$OACB$ is a parallelogram.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
P and Q are points on OC such that $OP = PQ = QC$.

(a) Express, as simply as possible, in terms of **a** and **b**,

(i) $\overrightarrow{OP}$,

Answer ..... [1]

(ii) $\overrightarrow{BP}$.

Answer ..... [1]

(b) Show that triangles OAQ and CBP are congruent.

[2]

Q21/11/M/J/18

4. $\mathbf{p} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

(a) Write $3\mathbf{p} - \mathbf{q}$ as a column vector.

Answer $\begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [1]

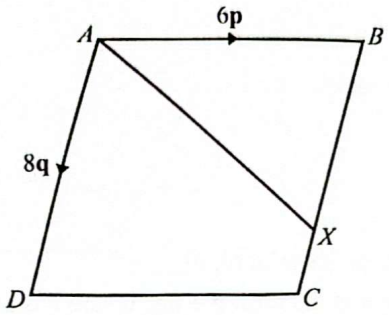
(b) R is the point $(11, -2)$ and O is the point $(0, 0)$.
The vector $\overrightarrow{OR}$ can be written in the form $\mathbf{p} + n\mathbf{q}$, where n is an integer.

Find the value of n.

Answer $n =$ ..... [2]

Q23/11/O/N/18

5.



In the diagram, $ABCD$ is a parallelogram.
X is the point on BC such that $BX : XC = 3 : 1$.
$\overrightarrow{AB} = 6\mathbf{p}$ and $\overrightarrow{AD} = 8\mathbf{q}$.

(a) Express $\overrightarrow{BX}$ in terms of $\mathbf{p}$ and/or $\mathbf{q}$.

Answer ..... [1]

(b) Express $\overrightarrow{AX}$ in terms of $\mathbf{p}$ and/or $\mathbf{q}$.

Answer ..... [1]

(c) Y is the point such that $\overrightarrow{CY} = 3\mathbf{p} + \mathbf{q}$.

(i) Express $\overrightarrow{AY}$ in terms of $\mathbf{p}$ and/or $\mathbf{q}$.

Answer ..... [1]

(ii) Find the ratio $AX : AY$.

Answer ..... : ..... [1]

Q23/12/O/N/18

6. **(a)** Express $\begin{pmatrix}2\\1\end{pmatrix}-3\begin{pmatrix}-1\\2\end{pmatrix}+2\begin{pmatrix}0\\-2\end{pmatrix}$ as a single vector.

Answer $\begin{pmatrix}\quad\\\quad\end{pmatrix}$ [2]

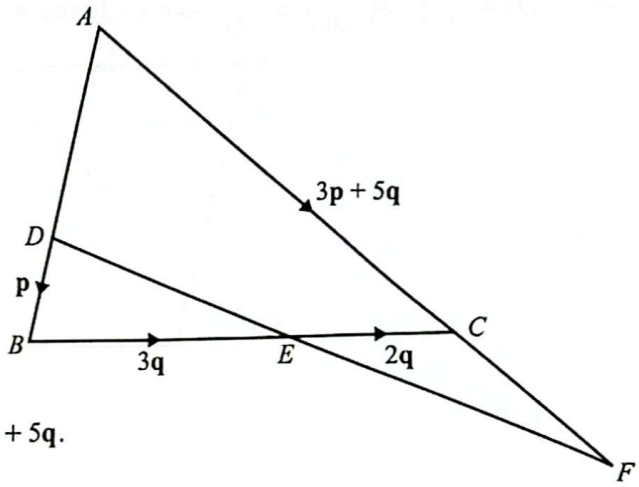
(b) Find $\begin{pmatrix}2 & -1\end{pmatrix}\begin{pmatrix}0 & -1 & 2\\3 & 1 & -3\end{pmatrix}$.

Answer [2]

Q25/12/O/N/18

7. In the diagram, ADB and ACF are straight lines.

BC intersects DF at E.



$AC : CF = 2 : 1$.

$\overrightarrow{DB} = \mathbf{p}$, $\overrightarrow{BE} = 3\mathbf{q}$, $\overrightarrow{EC} = 2\mathbf{q}$ and $\overrightarrow{AC} = 3\mathbf{p} + 5\mathbf{q}$.

(a) Express $\overrightarrow{AB}$ in terms of $\mathbf{p}$.

Answer $\overrightarrow{AB}$ = ... [1]

(b) Express $\overrightarrow{CF}$ in terms of $\mathbf{p}$ and/or $\mathbf{q}$.

Answer $\overrightarrow{CF}$ = ... [1]

(c) Express $\overrightarrow{EF}$ in terms of $\mathbf{p}$ and/or $\mathbf{q}$.

Answer $\overrightarrow{EF}$ = ... [1]

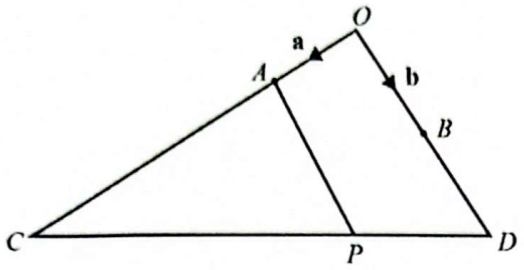
(d) $\overrightarrow{EF} = k\overrightarrow{DE}$.

Find k.

Answer k = ... [2]

Q25/11/O/N/19

8.



NOT TO SCALE

In the diagram, B is the midpoint of OD and $OA : AC = 1 : 3$.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Express, as simply as possible, in terms of **a** and/or **b**

(i) $\overrightarrow{OC}$,

$\overrightarrow{OC} =$ ..... [1]

(ii) $\overrightarrow{CD}$.

$\overrightarrow{CD} =$ ..... [1]

(b) P is the point on CD where $CP = \frac{3}{4}CD$.

(i) Express $\overrightarrow{AP}$, as simply as possible, in terms of **a** and/or **b**.

$\overrightarrow{AP} =$ ..... [2]

(ii) Find $AP : BD$.

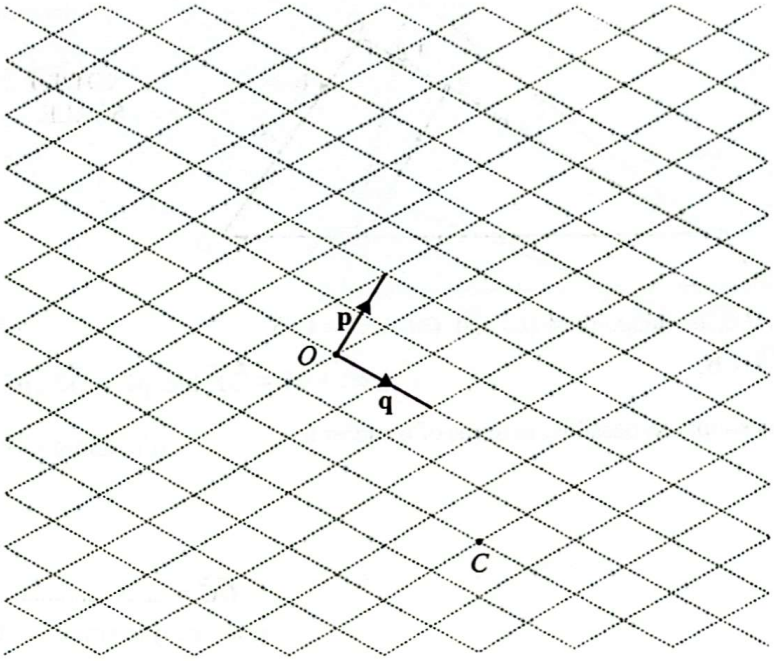
..... : ..... [1]

(iii) What special type of quadrilateral is $ABDP$?

..... [1]

Q21/12/O/N/19

9.



The diagram shows points O and C and the vectors $\mathbf{p}$ and $\mathbf{q}$.

(a) Given that $\overrightarrow{OA} = 2\mathbf{p}$, mark and label the point A on the diagram. [1]

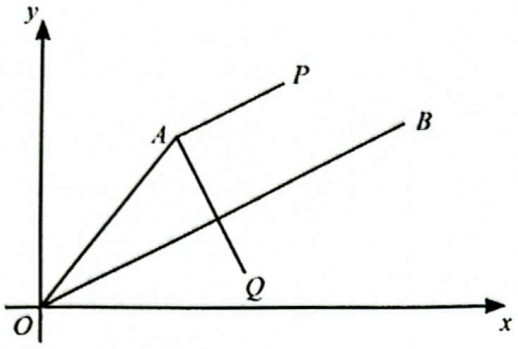
(b) Given that $\overrightarrow{OB} = \mathbf{p} - 2\mathbf{q}$, mark and label the point B on the diagram. [1]

(c) Express $\overrightarrow{OC}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

..... [2]

Q23/12/O/N/19

10.



NOT TO SCALE

In the diagram, $\overrightarrow{OB} = \begin{pmatrix} 12 \\ 6 \end{pmatrix}$.

(a) Show that the gradient of $OB = \frac{1}{2}$.

[1]

(b) AP is parallel to OB and $\overrightarrow{AP} = \begin{pmatrix} p \\ 2 \end{pmatrix}$.

(i) Find the value of p.

$p =$ ..... [1]

(ii) Write down the value of $\frac{AP}{OB}$.

..... [1]

(c) AQ is perpendicular to OB.

(i) Write down the gradient of AQ.

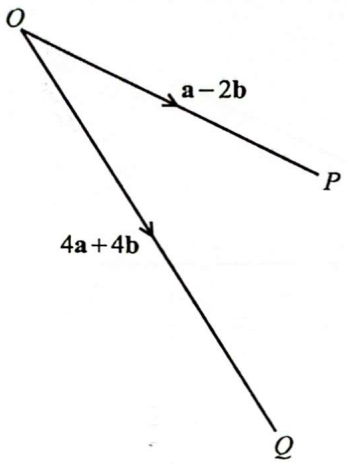
..... [1]

(ii) Given that $\overrightarrow{AQ} = \begin{pmatrix} 3 \\ q \end{pmatrix}$, find the value of q.

$q =$ ..... [1]

Q25/11/M/J/20

11. O, P and Q are points as shown in the diagram.



NOT TO SCALE

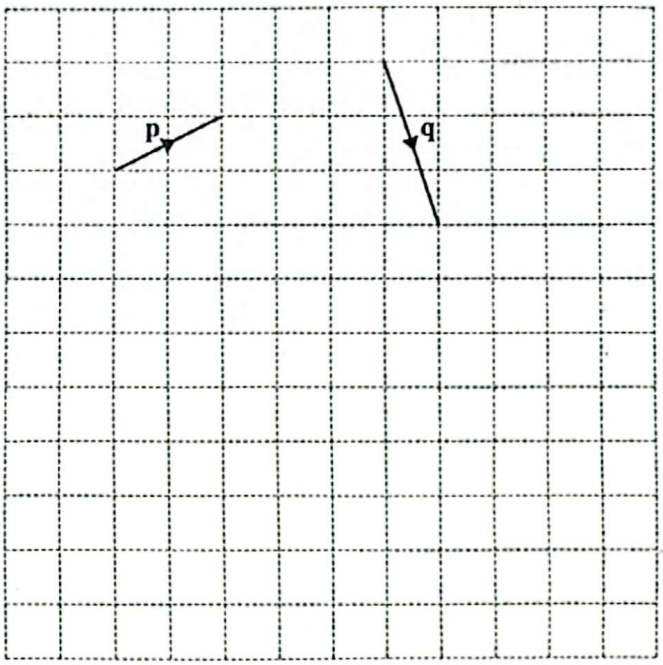
$\overrightarrow{OP} = \mathbf{a} - 2\mathbf{b}$ and $\overrightarrow{OQ} = 4\mathbf{a} + 4\mathbf{b}$.

Express $\overrightarrow{PQ}$, as simply as possible, in terms of **a** and **b**.

$\overrightarrow{PQ} =$ ..... [2]

Q19/12/M/J/20

12.



Vectors **p** and **q** are shown on the grid.

On the grid, draw the vector

(a) $3\mathbf{p}$, [1]

(b) $\mathbf{q}-\mathbf{p}$. [1]

Q15/11/O/N/21

13. The point A has position vector $\begin{pmatrix} 3 \\ -7 \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} -5 \\ 12 \end{pmatrix}$.

(a) Find the coordinates of point B.

(..... , ...) [2]

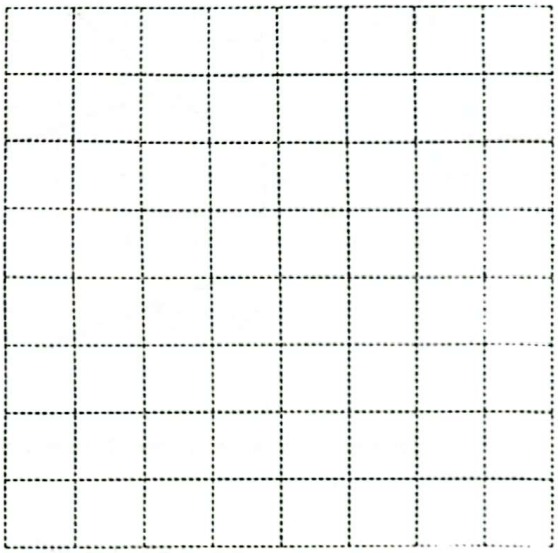
(b) Find $|\overrightarrow{AB}|$.

$|\overrightarrow{AB}|$ = ... units [2]

Q16/11/M/J/22

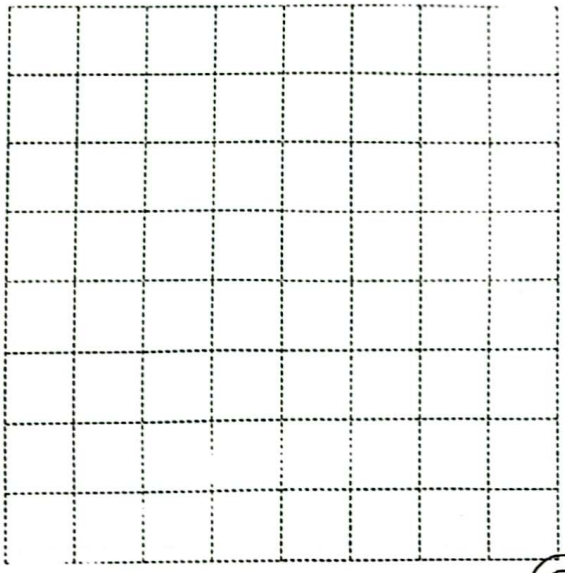
14. $\mathbf{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

(a) On the unit grid below, draw and label vector **p**.



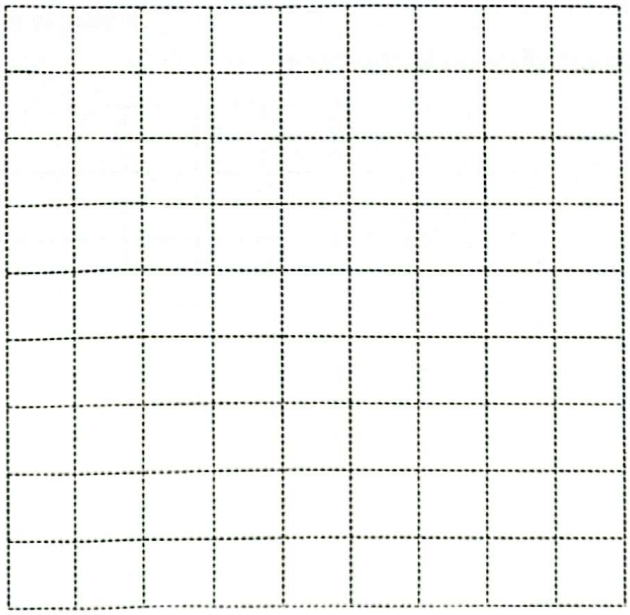
[1]

(b) On the unit grid below, draw and label vector 2**q**.



[1]

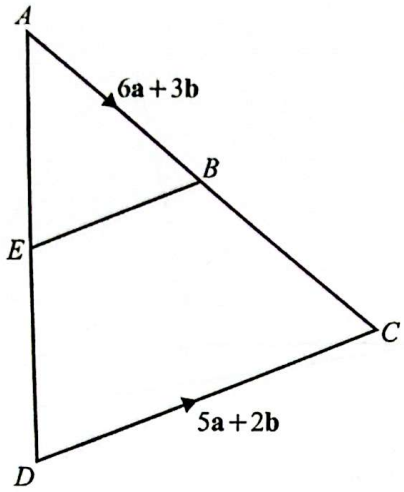
(c) On the unit grid below, draw and label vector $\mathbf{p}-\mathbf{q}$.



[2]

Q25/11/M/J/22

15.



NOT TO SCALE

In triangle ACD, B is the midpoint of AC and E is the midpoint of AD.
$\overrightarrow{AB} = 6\mathbf{a} + 3\mathbf{b}$ and $\overrightarrow{DC} = 5\mathbf{a} + 2\mathbf{b}$.

(a) Express, as simply as possible, in terms of **a** and **b**.

(i) $\overrightarrow{AC}$

$\overrightarrow{AC}$ = ... [1]

(ii) $\overrightarrow{AD}$

$\overrightarrow{AD}$ = ... [2]

(b) Show that $\overrightarrow{EB}$ is parallel to $\overrightarrow{DC}$.

.....

.....

.....

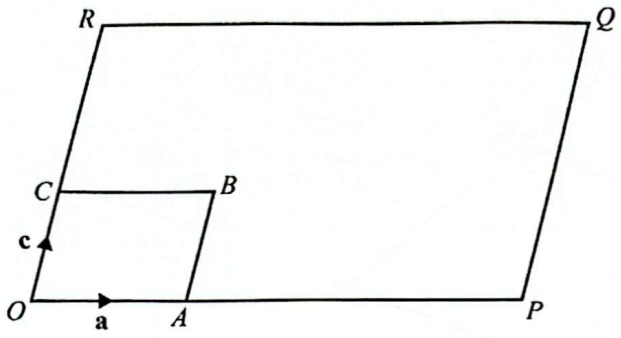
.....

.....

..... [3]

Q25/12/M/J/22

16.



NOT TO SCALE

$OABC$ and $OPQR$ are parallelograms.
A is a point on OP and C is a point on OR.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.
$OA : OP = 1 : 4$ and $OC : CR = 2 : 3$.

(a) Find $\overrightarrow{OR}$ in terms of **c**.

$\overrightarrow{OR}$ = ... [1]

(b) Find $\overrightarrow{CQ}$, as simply as possible, in terms of **a** and **c**.

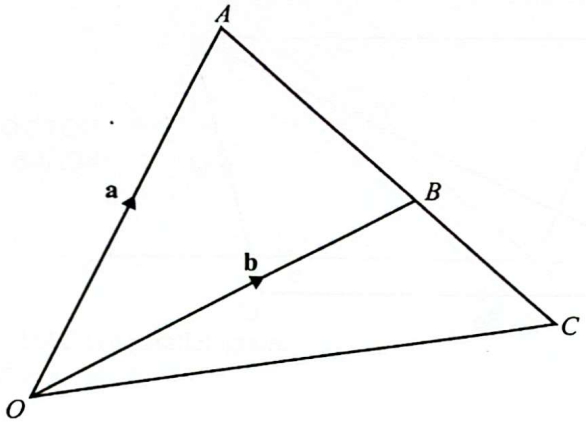
$\overrightarrow{CQ}$ = ... [2]

(c) Find the ratio area $OABC$: area $OPQR$.

..... : ..... [1]

Q26/12/O/N/22

17.



NOT TO SCALE

OAC is a triangle and B is a point on AC such that $AB : BC = 3 : 2$.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Find $\overrightarrow{OC}$ in terms of **a** and **b**, giving your answer in its simplest form.

$\overrightarrow{OC} =$ ..... [3]

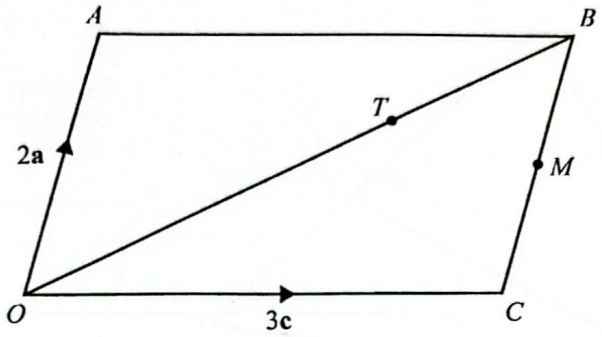
(b) D is a point on OC such that $\overrightarrow{OD} = \mathbf{b} - \frac{2}{5}\mathbf{a}$.

Show that $OABD$ is a trapezium.

[2]

Q27/11/M/J/23

18.



NOT TO SCALE

$OABC$ is a parallelogram.
$\overrightarrow{OA} = 2\mathbf{a}$ and $\overrightarrow{OC} = 3\mathbf{c}$.
M is the midpoint of BC.
T is the point on OB such that $OT : TB = 2 : 1$.

(a) Find $\overrightarrow{OB}$ in terms of **a** and **c**.

$\overrightarrow{OB} =$ ..... [1]

(b) Express, as simply as possible, in terms of **a** and **c**

(i) $\overrightarrow{AM}$

$\overrightarrow{AM} =$ ..... [1]

(ii) $\overrightarrow{AT}$.

$\overrightarrow{AT} =$ ..... [2]

(c) Show that ATM is a straight line.

.....

.....

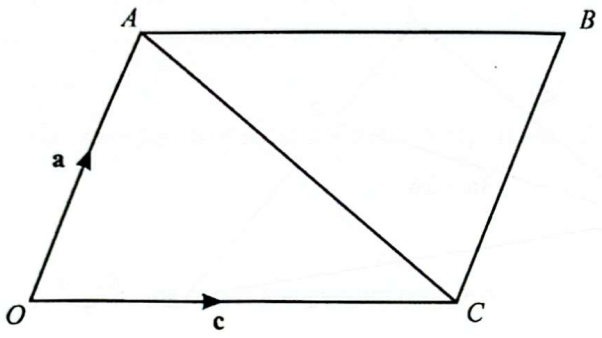
.....

.....

..... [2]

Q23/11/O/N/23

19.



NOT TO SCALE

In the diagram, $OABC$ is a parallelogram.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

X is the midpoint of AC.
Y is the point on AB where $AY : YB = 2 : 1$.

Express, as simply as possible, in terms of **a** and **c**

(a) $\overrightarrow{AC}$

$\overrightarrow{AC}$ = ... [1]

(b) the position vector of X

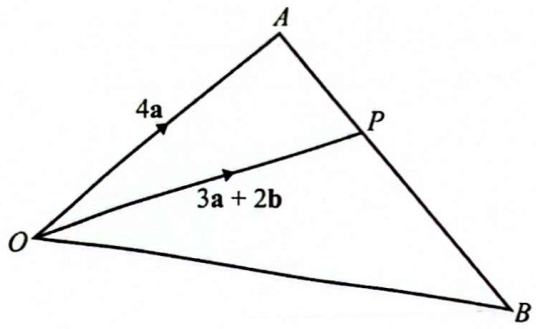
..... [2]

(c) $\overrightarrow{YX}$.

$\overrightarrow{YX}$ = ... [2]

Q24/12/O/N/23

20.



NOT TO SCALE

OAB is a triangle.
P lies on AB and $AP : PB = 2 : 3$.
$\overrightarrow{OA} = 4\mathbf{a}$ and $\overrightarrow{OP} = 3\mathbf{a} + 2\mathbf{b}$.

(a) Find, in terms of **a** and **b**, giving your answer in its simplest form

(i) $\overrightarrow{AP}$

$\overrightarrow{AP} =$ ..... [1]

(ii) $\overrightarrow{OB}$.

$\overrightarrow{OB} =$ ..... [3]

(b) Q is a point on OA such that $\overrightarrow{QP}$ is parallel to $\overrightarrow{OB}$.

Find $\overrightarrow{QP}$.

$\overrightarrow{QP} =$ ..... [1]

Q25/11/M/J/16

Question 1

Answers: **(a)(i) p – q (a)(ii) 3p – 4q (a)(iii) 9p – 9q (b) 1 : 8**

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Question 2

Answers: **(a)** Point *P* marked correctly **(b)** Point *Q* marked correctly **(c) -a – 2b**

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Question 3

Answers: **(a)(i)** $\frac{\mathbf{a}+\mathbf{b}}{3}$ **(ii)** $\frac{\mathbf{a}-2\mathbf{b}}{3}$ **(b)** shows triangles are congruent

Q21/11/M/J/18

Question 4

Answers: **(a)** $\begin{pmatrix}13\\9\end{pmatrix}$ **(b)** –2

Q23/11/O/N/18

Question 5

Answers: **(a) 6q (b) 6p + 6q (c)(i) 9p + 9q (ii)** 2:3

Q23/12/O/N/18

Question 6

Answers: **(a)** $\begin{pmatrix}5\\-9\end{pmatrix}$ **(b)** (–3 –3 7)

Q25/12/O/N/18

Question 7

Answers: **(a) 3p (b)** $\frac{1}{2}(3\mathbf{p}+5\mathbf{q})$ **(c)** $\frac{1}{2}(3\mathbf{p}+9\mathbf{q})$ **(d)** 1.5

Q25/11/O/N/19

Question 8

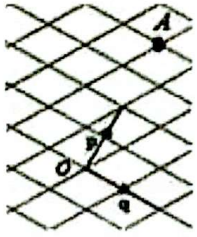
25(a)(i) **4a** 25(a)(ii) **2b – 4a**

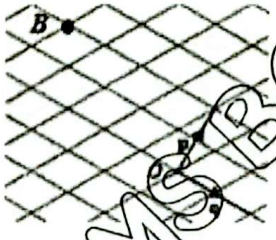
25(b)(i) $\frac{3}{2}\mathbf{b}$ 25(b)(ii) 3 : 2 oe 25(b)(iii) Trapezium

Q21/12/O/N/19

Question 9

21(a) *A* positioned correctly 21(b) *B* positioned correctly 21(c) **2q – p** oe





Q23/12/O/N/19

Question 10

23(a) Using gradient $= \frac{6}{12}$

23(b)(i) 4

23(b)(ii) $\frac{1}{3}$ oe

23(c)(i) –2

23(c)(ii) –6

Q25/11/M/J/20

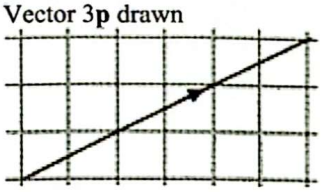
Question 11

25 $3\mathbf{a} + 6\mathbf{b}$

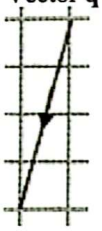
Q19/12/M/J/20

Question 12

19(a) Vector 3**p** drawn



19(b) Vector $\mathbf{q} - \mathbf{p}$ drawn



Q15/11/O/N/21

Question 13

15(a) (–2, 5)

15(b) 13

Q16/11/M/J/22

Question 14

16(a) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ drawn correctly with arrow

16(b) $\begin{pmatrix} -6 \\ 4 \end{pmatrix}$ drawn correctly with arrow

16(c) $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$ drawn correctly with arrow

Q25/11/M/J/22

Question 15

25(a)(i) $12\mathbf{a} + 6\mathbf{b}$ or $6(2\mathbf{a} + \mathbf{b})$ final answer

25(a)(ii) $7\mathbf{a} + 4\mathbf{b}$ final answer

25(b) $\overrightarrow{EB} = \overrightarrow{EA} + \overrightarrow{AB}$

or ½ *their* $\overrightarrow{DA} + 6\mathbf{a} + 3\mathbf{b}$
or $-$ ½ *their* $\overrightarrow{AD} + 6\mathbf{a} + 3\mathbf{b}$
or $-\frac{1}{2}(7\mathbf{a} + 4\mathbf{b}) + 6\mathbf{a} + 3\mathbf{b}$

$[\overrightarrow{EB} =] \ 2.5\mathbf{a} + \mathbf{b}$
or equivalent 2-term expression

$\overrightarrow{EB}$ is parallel to $\overrightarrow{DC}$ because
$\overrightarrow{EB} = k\overrightarrow{DC}$ oe

Q25/12/M/J/22

Question 16

25(a) $\frac{5}{2}\mathbf{c}$ oe final answer

25(b) $4\mathbf{a} + \frac{3}{2}\mathbf{c}$ or $\frac{8\mathbf{a} + 3\mathbf{c}}{2}$ final answer

25(c) 1 : 10 oe

Q26/12/O/N/22

Question 17

26(a) $\frac{5}{3}\mathbf{b} - \frac{2}{3}\mathbf{a}$ final answer

26(b) $\overrightarrow{DB} = \frac{2}{5}\mathbf{a}$ or $\overrightarrow{BD} = -\frac{2}{5}\mathbf{a}$

$\overrightarrow{DB}$ is a multiple of $\overrightarrow{OA}$, hence parallel oe

Q27/11/M/J/23

Question 18

27(a) $2\mathbf{a} + 3\mathbf{c}$ final answer

27(b)(i) $3\mathbf{c} - \mathbf{a}$ final answer

27(b)(ii) $2\mathbf{c} - \frac{2}{3}\mathbf{a}$ or $\frac{6\mathbf{c} - 2\mathbf{a}}{3}$ final answer

27(c) $\overrightarrow{AM} = \frac{3}{2}\overrightarrow{AT}$ oe and
AM is parallel to AT and
both go through A so ATM is a straight line
OR
$\overrightarrow{TM} = \mathbf{c} - \frac{1}{3}\mathbf{a}$ and
$\overrightarrow{AT} = 2\overrightarrow{TM}$ and
AT is parallel to TM and both go through T so
ATM is a straight line

Q23/11/O/N/23

Question 19

23(a) $\mathbf{c} - \mathbf{a}$

23(b) $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$ oe simplified vector

23(c) $-\frac{1}{2}\mathbf{a} - \frac{1}{6}\mathbf{c}$ oe simplified vector

Q24/12/O/N/23

Question 20

24(a)(i) $2\mathbf{b} - \mathbf{a}$ final answer

24(a)(ii) $\frac{3}{2}\mathbf{a} + 5\mathbf{b}$ final answer

or $\frac{3\mathbf{a} + 10\mathbf{b}}{2}$ final answer

24(b) $\frac{3}{5}\mathbf{a} + 2\mathbf{b}$ oe